7.7 Future Value of an Ordinary Annuity

When people invest, they usually do not simply deposit one lump sum and wait several years for it to earn interest. Most wise investors make regular payments, often deducted directly from their paycheques. Investments of this type are called annuities.

Define:

Annuity: - a series of equal deposits or payments made at regular intervals.

Ordinary Annuity: - is an annuity in which the payments are made at the end of each interval.

Simple (Regular) Annuity: - is an annuity in which the payments coincide with the compounding period.

All problems are at the end - Ordinary Simple Annuities.
****Payments and compound periods happen at the same time****

Example 1:

Kira deposits $100 at the end of each month into a savings account that earns 3%/a compounded monthly. What will her savings be at the end of 1 year?

Method 1: Use a time line diagram

End of Month Deposit	Calculation	Total
1	100	100.00
2	$100(1 + 0.03/12)^1$	100.25
3	$100(1 + 0.03/12)^2$	100.50
4	$100(1 + 0.03/12)^3$	100.75
5	$100(1 + 0.03/12)^4$	101.00
6	$100(1 + 0.03/12)^5$	101.26
7	$100(1 + 0.03/12)^6$	101.51
8	$100(1 + 0.03/12)^7$	101.76
9	$100(1 + 0.03/12)^8$	102.02
10	$100(1 + 0.03/12)^9$	102.27
11	$100(1 + 0.03/12)^{10}$	102.53
12	$100(1 + 0.03/12)^{11}$	102.78
	Total	1216.64

Method 2: Formula

where:

$$A = \frac{R[(1+i)^n - 1]}{i}$$

A is the future Amount in $
R is the regular deposit
i is interest rate per period
n is total the number of deposits
# of years X compounding periods

$$= 100\frac{\left(\left(1+\frac{0.03}{12}\right)^{12}-1\right)}{\left(\frac{0.03}{12}\right)}$$

$$= 1216.64$$

∴ she will have $1216.64 after 1 year.

Examples:

$$A = \frac{R[(1+i)^n - 1]}{i}$$

Determine the amount of each annuity.

a) Regular deposits of $500 every 6 months for 4 years at 8%/a compounded semi-annually.

$$= 500\left(\frac{\left(1+\frac{0.08}{2}\right)^8 - 1}{\left(\frac{0.08}{2}\right)}\right) \quad \text{same} \quad = 4607.11$$

b) Regular deposits of $200 every month for 8 years at 10%/a compounded monthly.

$$= 200\left(\frac{\left(1+\frac{0.10}{12}\right)^{96} - 1}{\left(\frac{0.10}{12}\right)}\right)$$

$$= 29236.22$$

HMWK:
p 498 # 4, 5, 7, 10, 12

***** Multi Step*****
 # 2, 9, 11, 17